

# C.U.SHAH UNIVERSITY

## Winter Examination-2018

**Subject Name : Engineering Mathematics - II**

**Subject Code : 4TE02EMT3**

**Branch: B. Tech (All)**

**Semester : 2**

**Date : 23/10/2018**

**Time : 02:30 To 05:30**

**Marks : 70**

**Instructions:**

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1 Attempt the following questions:**

**(14)**

a) The interval of convergence of the logarithmic series

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \infty \text{ is}$$

(A)  $-1 < x \leq 1$  (B)  $-1 < x < 2$  (C)  $-\infty < x < \infty$  (D)  $-1 \leq x \leq 1$

b) The infinite series  $1 + r + r^2 + \dots + r^{n-1}$  is divergent if

(A)  $|r| < 1$  (B)  $|r| > 1$  (C)  $r \geq 1$  (D)  $r = -1$

c)  $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x \, dx$  is equal to

(A)  $\frac{1}{16}$  (B)  $\frac{1}{32}$  (C)  $\frac{\pi}{4}$  (D)  $\frac{\pi}{32}$

d) Let  $f(b)$  be an odd function in the interval  $\left[-\frac{T}{2}, \frac{T}{2}\right]$  with a period  $T$ , then

$$F(x) = \int_a^x f(t) \, dt \text{ is}$$

(A) periodic (B) non-periodic (C) periodic with period  $2T$   
(D) periodic with period  $4T$

e) If  $f$  is continuous, decreasing function on  $[1, \infty)$  and  $\lim_{x \rightarrow \infty} f(x) = 0$ , then

$$\int_1^{\infty} f(x) \, dx \text{ is convergent.}$$

(A) True (B) False

f)  $\sqrt{4.5} =$  \_\_\_\_\_

(A)  $\frac{\sqrt{\pi}}{16}$  (B)  $\frac{105\sqrt{\pi}}{16}$  (C)  $\frac{5\sqrt{\pi}}{16}$  (D) none of these



- g) Duplication formula:  $\sqrt[n]{n + \frac{1}{2}} = \underline{\hspace{2cm}}$   
 (A)  $\frac{\sqrt{\pi} \sqrt{n}}{2^{2n-1}}$  (B)  $\frac{\sqrt{\pi} \sqrt{2n}}{2^{n-1}}$  (C)  $\frac{\sqrt{\pi} \sqrt{2n}}{2^{2n-1}}$  (D)  $\frac{\sqrt{\pi} \sqrt{n}}{2^{n-1}}$
- h) If the two tangents at the point are real and distinct the double point is called  
 (A) a node (B) a cusp (C) a conjugate point (D) none of these
- i)  $\int_0^1 \int_0^x e^x dx dy$  is equal to  
 (A)  $-1$  (B)  $1$  (C)  $e$  (D)  $e^{-1}$
- j) The transformations  $x + y = u, y = uv$  transform the area element  $dy dx$  into  $|J| du dv$ , where  $|J|$  is equal to  
 (A)  $1$  (B)  $u$  (C)  $-1$  (D) none of these
- k)  $\int_0^2 \int_1^3 \int_1^2 xy^2z dz dy dx = \underline{\hspace{2cm}}$   
 (A)  $62$  (B)  $26$  (C)  $24$  (D)  $42$
- l) The degree of the differential equation  $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x \log\left(\frac{d^2y}{dx^2}\right)$  is  
 (A)  $1$  (B)  $2$  (C)  $3$  (D) none of these
- m) If  $\frac{dy}{dx} + \frac{1}{y\sqrt{1-x^2}} = 0$ , then which of the following statements is true?  
 (A)  $y + \sin^{-1} x = 0$  (B)  $y^2 + 2\sin^{-1} x = c$  (C)  $x + \sin^{-1} y = c$  (D)  $y = k$
- n) The homogeneous differential equation  $f_1(x, y)dx + f_2(x, y)dy = 0$  can be reduced to a differential equation in which the variables are separated, by the substitution  
 (A)  $y = vx$  (B)  $x + y = v$  (C)  $xy = v$  (D)  $x - y = v$

**Attempt any four questions from Q-2 to Q-8**

**Q-2**

**Attempt all questions**

**(14)**

- a) Show that  $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$  is (i) convergent if  $p > 1$  and (ii) divergent if  $p \leq 1$ .

**(5)**

- b) Using reduction formula evaluate  $\int_0^{\frac{1}{2}} x^3 \sqrt{1-4x^2} dx$ .

**(5)**

- c) Prove that  $\int_0^2 x^4 (8-x^3)^{\frac{1}{3}} dx = \frac{16}{3} B\left(\frac{5}{3}, \frac{2}{3}\right)$ .

**(4)**

**Q-3**

**Attempt all questions**

**(14)**

- a) Evaluate:  $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx$

**(5)**

- b) Using reduction formula prove that  $\int_0^a x^5 (2a^2 - x^2)^{-3} dx = \frac{1}{2} \left(\log 2 - \frac{1}{2}\right)$ .

**(5)**



c) Discuss the convergence of  $\sum \frac{\sqrt{n+1} - \sqrt{n}}{n}$ . (4)

**Q-4 Attempt all questions** (14)

a) Change the order of integration in  $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx dy$  and evaluate it. (5)

b) Test for convergence the series  $2 + \frac{3}{2}x + \frac{4}{3}x^2 + \frac{5}{4}x^3 + \dots (x > 0)$  by ratio test. (5)

c) Solve:  $\frac{dy}{dx} = \cos x \cos y - \sin x \sin y$  (4)

**Q-5 Attempt all questions** (14)

a) Solve:  $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$  (5)

b) Evaluate the double integral  $\iint_R (x^2 + y^2) dx dy$ , where R is the square bounded by lines  $y = x$ ,  $y = -x$ ,  $x - y = 2$ ,  $x + y = 2$  using transformations,  $u = x + y$  and  $v = x - y$ . (5)

c) Using reduction formula, evaluate  $\int_0^{\frac{\pi}{6}} \cos^6 3\theta \sin^2 6\theta d\theta$ . (4)

**Q-6 Attempt all questions** (14)

a) Prove that  $\int_{-\infty}^{\infty} e^{-k^2 x^2} dx = \frac{\sqrt{\pi}}{k}$ . (5)

b) Solve:  $(x^2 - y^2) dy = 2xy dx$  (5)

c) Evaluate:  $\int_0^{\frac{\pi}{2}} \int_0^{a \sin \theta} \int_0^a r dr d\theta dz$  (4)

**Q-7 Attempt all questions** (14)

a) Find the asymptotes of the curve  $y^3 - x^2(6-x) = 0$ . (5)

b) Find the area of the region outside the circle  $r = 2$  and inside the lemniscate  $r^2 = 8 \cos 2\theta$ . (5)

c) Investigate the convergence of  $\int_2^5 \frac{1}{\sqrt{(x-2)}} dx$ . (4)

**Q-8 Attempt all questions** (14)

a) Evaluate:  $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$  (5)

b) Trace the curve  $r = a(1 + \cos \theta)$ . (5)

c) Find the length of the arc of the Catenary  $y = c \cosh\left(\frac{x}{c}\right)$  measured from the vertex  $(0, c)$  to any point on the Catenary. (4)

